UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level/

MATHEMATICS (US)   **9280/03**

Paper 3 Pure Mathematics 3 **(P3)**   **For Examination from 2013**

SPECIMEN MARK SCHEME

1 hour 45 minutes

MAXIMUM MARK: 75

This document consists of **7** printed pages and **1** blank page.

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Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips, or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g., by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g., wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained that is correct to 3 s.f., or that would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers that arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasizing that no "follow through" from a previous error is allowed)

CWO   Correct Working Only often written by a "fortuitous" answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads their own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1   *EITHER* Obtain $1+\frac{1}{3}kx$ where $k=\pm6$ or ±1

Obtain $1-2x$

Obtain $-4x^2$ A1

Obtain $-\frac{40}{3}x^3$ or equivalent   A1

OR   Differentiate expression to obtain form $k(1-6x)^{-\frac{2}{3}}$ and evaluate f(0) and f'(0)   M1

Obtain f'$(x)=-2(1-6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1-2x$ A1

Obtain f''$(x)=-8(1-6x)^{-\frac{5}{3}}$ and hence $-4x^2$ A1

Obtain f'''$(x)=-80(1-6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent   A1   [4]

2   **(i)**   Obtain $\dfrac{k\cos 2x}{1+\sin 2x}$ for any non-zero constant k M1

Obtain $\dfrac{2\cos 2x}{1+\sin 2x}$ A1   [2]

(ii) Use correct quotient or product rule   M1

Obtain $\dfrac{x\sec^2 x-\tan x}{x^2}$ or equivalent   A1   [2]

3   **(i)**   Obtain $\pm\begin{pmatrix}3\\-4\\6\end{pmatrix}$ as normal to plane   B1

Form equation of p as $3x-4y+6z=k$ or $-3x+4y-6z=k$ and use relevant point to find k M1
Obtain $3x-4y+6z=80$ or $-3x+4y-6z=-80$ A1   [3]

(ii) State the direction vector $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or equivalent   B1

Carry out correct process for finding scalar product of two relevant vectors   M1
Use correct complete process with moduli and scalar product and evaluate $\sin^{-1}$ or $\cos^{-1}$
of result   M1
Obtain 30.8° or 0.538 radians   A1   [4]

4 (i) Verify that $-96 + 100 + 8 - 12 = 0$

Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity   M1

Obtain $12x^2 + x - 6$ A1

State $(x + 2)(4x + 3)(3x - 2)$ A1   [

[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]

(ii) State $3^y = \frac{2}{3}$ and no other value   B1

Use correct method for finding y from equation of form $3^y = k$, where $k > 0$ M1

Obtain -0.369 and no other value   A1   [3]

5 (i) Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$ B1

Obtain given result $58 + 2k = c$ **AG**   B1   [2]

(ii) Differentiate left-hand side term by term, reaching $ae^{2x} + be^y \dfrac{dy}{dx} + ce^{2y} \dfrac{dy}{dx}$ M1

Obtain $12e^{2x} + ke^y \dfrac{dy}{dx} + 2e^{2y} \dfrac{dy}{dx}$ A1

Substitute $(\ln 3, \ln 2)$ in an attempt involving implicit differentiation at least once, where RHS $= 0$ M1

Obtain $108 - 12k - 48 = 0$ or equivalent   A1

Obtain $k = 5$ and $c = 68$ A1   [5]

6 (i) State or imply area of segment is $\frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta$ or $50\theta - 50\sin\theta$ B1

Attempt to form equation from area of segment $= \frac{1}{5}$ of area of circle, or equivalent   M1

Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$ A1   [3]

(ii) Use iterative formula correctly at least once   M1

Obtain value for θ of 2.11   A1

Show sufficient iterations to justify value of θ or show sign change in interval $(2.105, 2.115)$ A1

Use correct trigonometry to find an expression for the length of AB M1

e.g., $20\sin 1.055$ or $\sqrt{200 - 200\cos 2.11}$

Hence 17.4   A1   [5]

$[2.1 \rightarrow 2.1198 \rightarrow 2.1097 \rightarrow 2.1149 \rightarrow 2.1122]$

7 **(i)** State or imply $dx = 2t \, dt$ or equivalent

Express the integral in terms of x and dx

Obtain given answer $\int_1^5 (2x - 2) \ln x \, dx$, including change of limits **AG**   A1

(ii) Attempt integration by parts obtaining $(ax^2 + bx)\ln x \pm \int (ax^2 + bx) \frac{1}{x} dx$ or equivalent   M1

Obtain $(x^2 - 2x)\ln x - \int (x^2 - 2x) \frac{1}{x} dx$ or equivalent   A1

Obtain $(x^2 - 2x)\ln x - \frac{1}{2} x^2 + 2x$ A1

Use limits correctly having integrated twice   M1

Obtain $15 \ln 5 - 4$ or exact equivalent   A1   [5]

[Equivalent for M1 is $(2x - 2)(ax \ln x + bx) - \int (ax \ln x + bx) \, 2dx$]

8 **(i)** *EITHER* Multiply numerator and denominator by $(1 - 2i)$, or equivalent   M1

Obtain $-3i$ A1

State modulus is 3   A1

Refer to u being on negative imaginary axis or equivalent and confirm argument

as $-\frac{1}{2}\pi$ A1

OR Using correct processes, divide moduli of numerator and denominator   M1

Obtain 3   A1

Subtract argument of denominator from argument of numerator   M1

Obtain $-\tan^{-1} \frac{1}{2} - \tan^{-1} 2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$ or -1.57 A1   [4]

(ii) Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction   B1

Use correct trigonometry to find required value   M1

Obtain $\frac{3}{2}\sqrt{2}$ or equivalent   A1   [3]

(iii) Show, or imply, locus is a circle with center $(1 + i)u$ and radius 1   M1

Use correct method to find distance from origin to furthest point of circle   M1

Obtain $3\sqrt{2} + 1$ or equivalent   A1   [3]

9 **(i)** Express $\cos 4\theta$ as $2 \cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2 \sin^2 2\theta$

Express $\cos 4\theta$ in terms of $\cos \theta$

Obtain $8 \cos^4 \theta - 8 \cos^2 \theta + 1$ A

Use $\cos 2\theta = 2 \cos^2 \theta - 1$ to obtain given answer $8 \cos^4 \theta - 3$ **AG**   A1

(ii) (a) State or imply $\cos^4 \theta = \frac{1}{2}$ B1

Obtain 0.572   B1

Obtain –0.572   B1   [3]

(b) Integrate and obtain form $k_1 \theta + k_2 \sin 4\theta + k_3 \sin 2\theta$ M1

Obtain $\frac{3}{8}\theta + \frac{1}{32} \sin 4\theta + \frac{1}{4} \sin 2\theta$ A1

Obtain $\frac{3}{32}\pi + \frac{1}{4}$ following completely correct work   A1   [3]

10 **(i)** Separate variables correctly and integrate at least one side   M1

Carry out an attempt to find A and B such that $\dfrac{1}{N(1800 - N)} \equiv \dfrac{A}{N} + \dfrac{B}{1800 - N}$, or equivalent   M1

Obtain $\dfrac{2}{N} + \dfrac{2}{1800 - N}$ or equivalent   A1

Integrates to produce two terms involving natural logarithms   M1

Obtain $2 \ln N - 2 \ln (1800 - N) = t$ or equivalent   A1

Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a \ln N$, $b \ln(1800)$

and ct M1

Obtain $2 \ln N - 2 \ln (1800 - N) = t - 2 \ln 5$ or equivalent   A1

Use laws of logarithms to remove logarithms   M1

Obtain $N = \dfrac{1800 e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent   A1   [9]

(ii) State or imply that N approaches 1800   B1   [1]

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